Using Fuzzy Logic to Model Risk Case Studies

Marie-Claire Koissi
University of Wisconsin – Eau Claire

Arnold F. Shapiro
Pennsylvania State University

***

Thanks to
CAS/CIA/SOA
Joint Risk Management Section

Introduction

- **Risk assessment** (RA) is a systematic process for identifying and evaluating potential risks and opportunities that could positively or negatively affect the achievement of an enterprise's objectives.

- **The role of FL as it relates to RA:**

Fuzzy logic (FL) allows qualitative knowledge about a problem to be translated into an executable rule set.

In terms of risk modeling and assessment, fuzzy logic shows potential to be a good approach in dealing with operational risk, where the probability assessment is often based on expert opinion …." [CAS (2003: 42)]

Introduction

- **Fuzzy Numbers** (FN)

Examples of fuzzy numbers are the notions of “around four percent” and “relatively low”.

The general characteristic of a fuzzy number is its **membership function** that assigns to each object a grade of membership ranging between zero and one.

An example: membership function (MF), $\mu_{high}(x)$.

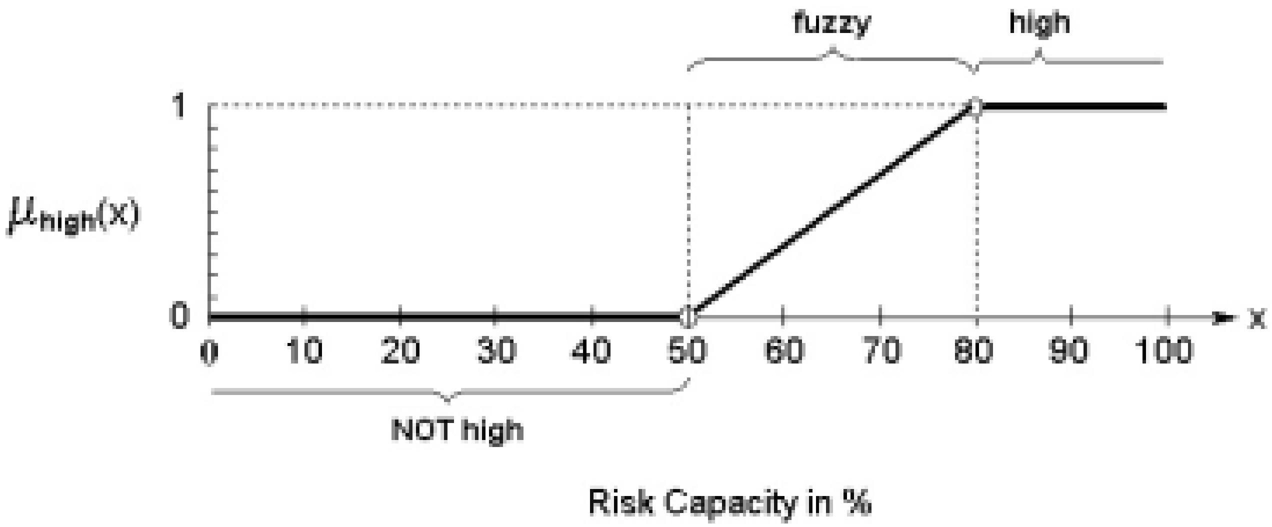
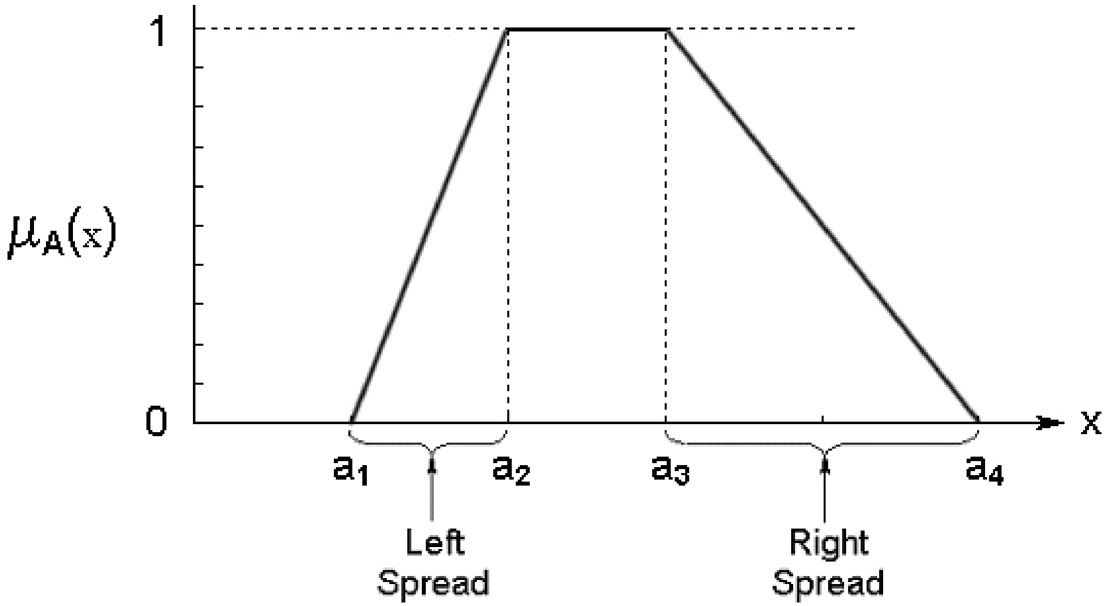


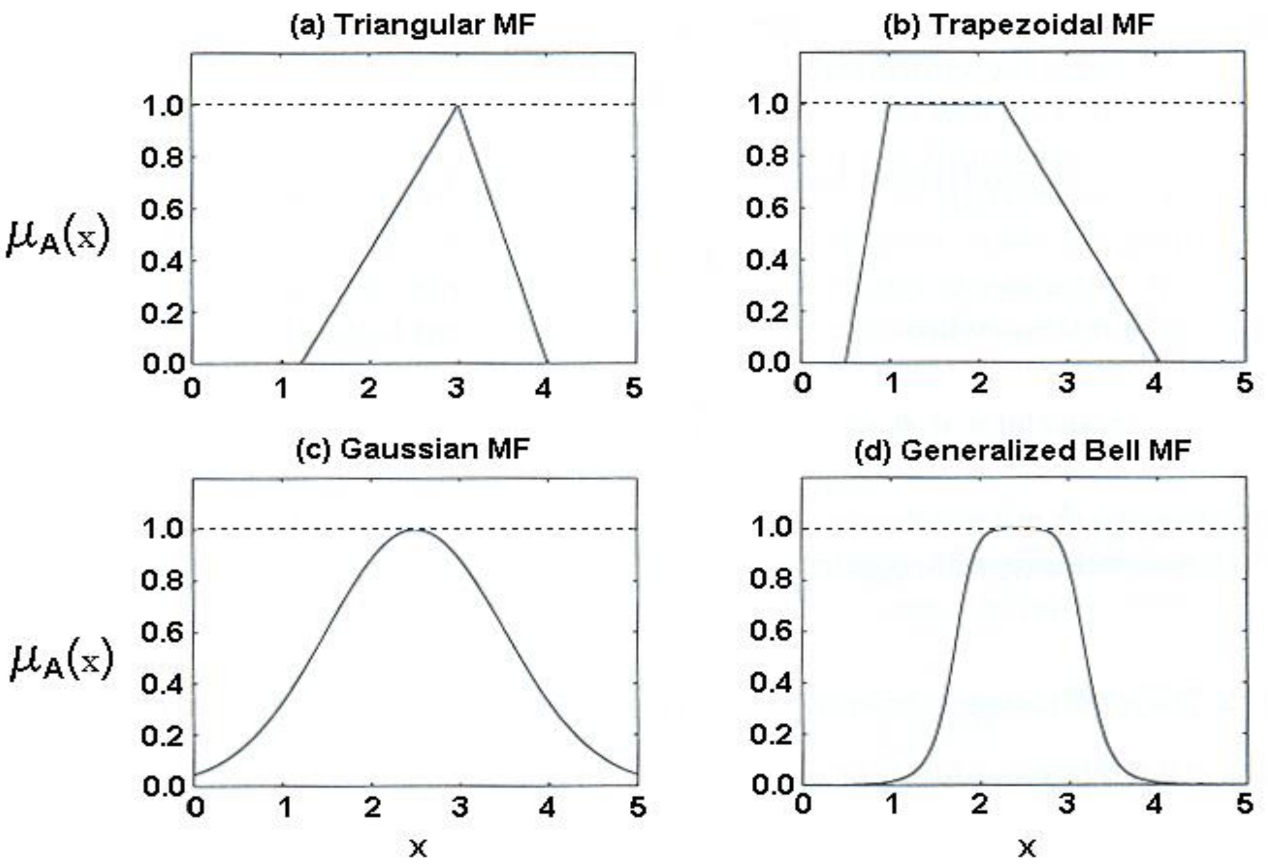
Figure 1: (Fuzzy) Set of Clients with High Risk Capacity

Introduction

- Fuzzy Numbers (FN)
  Other forms of **membership function**



Trapezoidal or flat Fuzzy Number
(a_1, a_2, a_3, a_4) or $(a_1/a_2, a_3/a_4)$



Examples of Classes of MFs

Cases Study 1: Using FL for Operational Risk Assessment: A Scenario Analysis Example

- **The operational risk** refers to unexpected changes associated with operations, such as human resources, technology, processes, and disasters (Segal, 2011, p. 116).
- In particular, the operational risk is "the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events" (Basel II).

- **Scenario analysis** aims at providing decision makers with experts' opinions on the company exposure in case of for example rare operational risk events that have high-severity losses.

Cases Study 1: Using Fuzzy Logic for Operational Risk Assessment: A Scenario Analysis Example

- This example is adapted from the paper by Durfee and Tselykh (2011)

Authors: Durfee (Citizens Bank), and Tselykh (South. Fed. Univ)

Paper: “Evaluating Operational Risk Exposure Using Fuzzy Number Approach to Scenario Analysis”, EUSFLAT-LFA, Atlantis Press, France.

- In this study, expert opinion is solicited to describe a potential large and plausible extreme operational risk loss event. Each team of experts is asked to produce optimistic, realistic, and pessimistic estimates for severity and frequency for each scenario storylines.

Cases Study 1: Using FL for Operational Risk Assessment: A Scenario Analysis Example

- Pessimistic, realistic, and optimistic experts’ judgments
  - on financial impact of scenario are denoted by $[a_{min}, a_{mod}, a_{max}]$
  - on the frequency of scenario is given in the form $[b_{min}, b_{mod}, b_{max}]$.

Using FL for Operational Risk Assessment: A Scenario Analysis Example

- **Flowchart of methodology used in this example**

Experts' scenario on severity
$(a_{\min}, a_{\text{mod}}, a_{\max})$

↓

LR- triangular fuzzy representation of severity
$(a_{\text{mod}}, a_{\text{mod}} - a_{\min}, a_{\max} - a_{\text{mod}}) = (c_S, l_S, r_S)$

↓

Experts' scenario on frequency of the event
$(b_{\min}, b_{\text{mod}}, b_{\max})$

↓

LR- triangular fuzzy representation of frequency
$(1/b_{\text{mod}}, 1/b_{\text{mod}} - 1/b_{\max}, 1/b_{\min} - 1/b_{\text{mod}}) = (c_P, l_P, r_P)$

↓

Computing the operational risk
$S \otimes P(S) = (c_S, l_S, r_S) \otimes (c_P, l_P, r_P)$

- Scenario data on **financial impact of loss event (severity)**

Events	Type of Scenario	Severity (S)			Fuzzy representation LR-TFN
		Optimistic $a_{\min}$	Realistic a_{mod}	Pessimistic $a_{\max}$	
1	External fraud (branch robberies)	\$ 1.2 million	\$ 1.7 million	\$ 2.2 million	(1.7, 0.5, 0.5)
2	Regulatory breaches (can cause large fines and costly law suites)	\$ 1.5 million	\$ 3.5 million	\$ 5.0 million	(3.5, 2.0, 1.5)

$$A = (a_{mod},\ a_{mod} - a_{min},\ a_{mod} - a_{max}) = (1.7,\ 1.7 - 1.2,\ 2.2 - 1.7) = (1.7, 0.5, 0.5) \quad \text{for scenario 1}$$

$$= (3.5,\ 3.5 - 1.5,\ 5.0 - 3.5) = (3.5, 2.0, 1.5) \quad \text{for scenario 2}$$

- Scenario data on **frequency of loss event**

Events	Type of Scenario	Frequency (P)			Fuzzy representation LR-TFN
		Optimistic a_{min}	Realistic a_{mod}	Pessimistic a_{max}	
1	External fraud (branch robberies) Happen often, with low financial impact.	5 (once every 5 years)	10 (once every 10 years)	15 (once in 15 years)	(1/10 , 1/ 30, 1/10)
2	Regulatory breaches (Happen less frequently, but severe financially)	4 (once every 4 years)	8 (once every 8 years)	12 (once every 12 years)	(1/8 , 1/ 24, 1/8)

Scenario 1:

[bmin ,bmod ,bmax] = [5 , 10 , 15] , and P(S) = (1/bmax, 1/bmod , 1/bmin) = (1/15, 1/10, 1/5)

The corresponding LR-type triangular fuzzy number is

LR - P(S) = (1/bmod , 1/bmod -1/bmax, 1/bmin - 1/bmod) = (1/10 , 1/10 – 1/15, 1/5 – 1/10)

= (1/10 , 1/ 30, 1/10)

Computing the operational risk (OR): "Product" of frequency and severity

$$\text{OR} = \text{S} \otimes \text{P(S)} = (c_S,\ l_S,\ r_S) \otimes (c_P,\ l_P,\ r_P)$$

$$= (c_S \,.\, c_P,\ \ c_S\, r_P + c_P\, r_S - r_S\, r_P,\ \ c_S\, l_P + c_P\, l_S + l_S\, l_P)$$

Events	Type of Scenario	Fuzzy representation LR-TFN (Note: numbers are in millions)		Operational risk (OR) exposure LR-TFN
		Frequency $(c_P,\ l_P,\ r_P)$	Severity $(c_S,\ l_S,\ r_S)$	
1	External fraud (branch robberies for example). Happen often, with low financial impact.	(1/10 , 1/30, 1/10)	(1.7, 0.5, 0.5)	(0.1700, 0.0900, 0.2700)
2	Regulatory breaches (Happen less frequently, but severe financially)	(1/8 , 1/ 24, 1/8)	(3.5, 2.0, 1.5)	(0.1944, 0.3125, 0.8125)

This is an approximation as shown in the figure below which also shows the relationship between exact fuzzy multiplication and the approximation used.

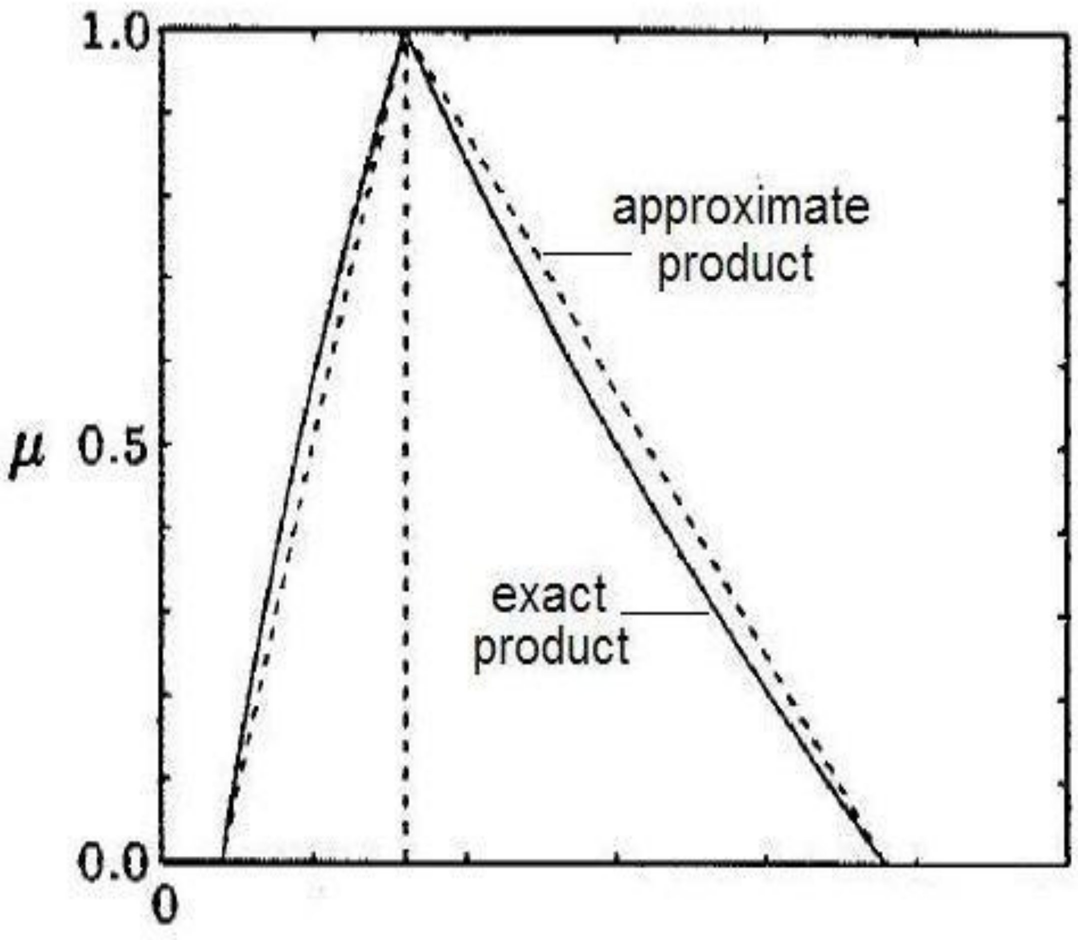


Figure: Exact vs. approximate fuzzy multiplication

Fuzzy multiplication:

$\mu_C (z) = \max [\min (\mu_A (x), \mu_B (y)), \ x * y = z]$

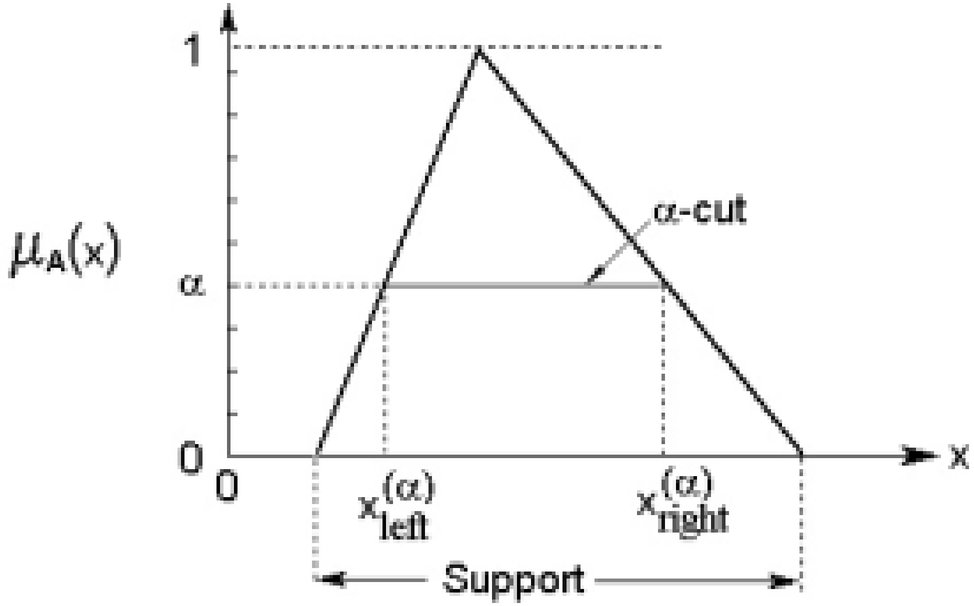
The α- cut of the operational risk

The degree of confidence with which an **expert makes a judgment** correspond to a notion of α- cut of a fuzzy number.

$$OR_{\alpha} = (c_{OR},\ l_{OR},\ r_{OR})_{\alpha} = (c_{OR} - (1- \alpha)\, l_{OR},\quad c_{OR} + (1- \alpha)\, r_{OR})$$

Let A be a fuzzy set in the universe X. The (crisp) set of elements that belong to the fuzzy set A at least to the degree α is called the α-cut or α-level set and is defined by:

$$A_{\alpha}\{x \in X \mid \mu_A(x) \geq \alpha\}$$



The α- cut of the operational risk

$OR_{\alpha} = (c_{OR}, l_{OR}, r_{OR})_{\alpha} = (c_{OR} - (1-\alpha)\, l_{OR}, \quad c_{OR} + (1-\alpha)\, r_{OR})$

Scenario 1:

$OR_{\alpha} = (0.1700, 0.0900, 0.2700)_{\alpha} = (0.17 - (1-\alpha)\, 0.09, \quad 0.17 + (1-\alpha)\, 0.27)$

$= (0.1565, 0.2105)$ for $\alpha = 0.85$

$= (0.1655, 0.1835)$ for $\alpha = 0.95$

Events	Type of Scenario	Fuzzy representation LR-TFN (Note: numbers are in millions)		Operational risk (OR) exposure LR-TFN	Degree of confidence	
		Frequency (c_P, l_P, r_P)	Severity (c_S, l_S, r_S)		$\alpha = 0.85$	$\alpha = 0.95$
1	External fraud	(1/10 , 1/30, 1/10)	(1.7, 0.5, 0.5)	(0.1700, 0.0900, 0.2700)	(0.1565, 0.2105)	(0.1655, 0.1835)
2	Regulatory breaches	(1/8 , 1/ 24, 1/8)	(3.5, 2.0, 1.5)	(0.1944, 0.3125, 0.8125)	(0.1475, 0.3163)	(0.1788, 0.2350)

The α- cut of the operational risk

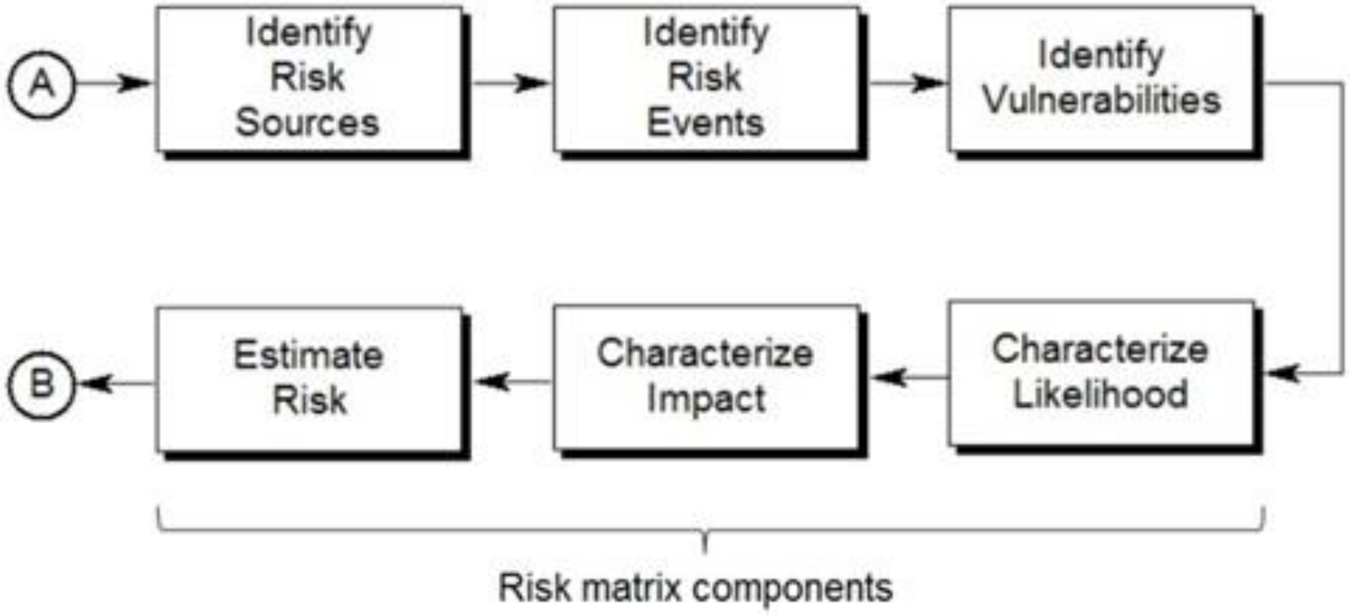
$$OR_{\alpha} = (c_{OR} - (1-\alpha)\, l_{OR}, \quad c_{OR} + (1-\alpha)\, r_{OR})$$

Adding operational risk for two or more scenarios: **α-** cut are added.

$$(\text{sum})\, OR_{\alpha} = (\, (\text{sum})\, c_{OR} - (1-\alpha)\, (\text{sum})\, l_{OR},$$
$$(\text{sum})\, c_{OR} + (1-\alpha)\, (\text{sum})\, r_{OR}\,)$$

Using Fuzzy Logic for Risk Assessment: Common Steps

- Cases Study 2: Using Fuzzy Logic for Hazard Risk Assessment: A Health Risk Example
- Cases Study 3: Using Fuzzy Logic for Financial Risk Assessment: An Example of portfolio Cash Risk Assessment
- Cases study 4: Using Fuzzy Logic for Strategic Risk Assessment: An Investment Case Selection



Short Note on the (Fuzzy) Risk Matrix

Severity		Likelihood 1	Likelihood 2	Likelihood 3	Likelihood 4	Likelihood 5
	5	MH 5	MH	MH	H	H 25
	4	M	M	MH	MH	H
Severity	3	M	M 8	M	MH	MH
	2	L	L 7	L	M	MH
	1	L 1	L	L	M	MH

Likelihood: 1 2 3 4 5

Risk categories:
1. low (L)
2. medium (M)
3. medium high (MH)
4. high (H)

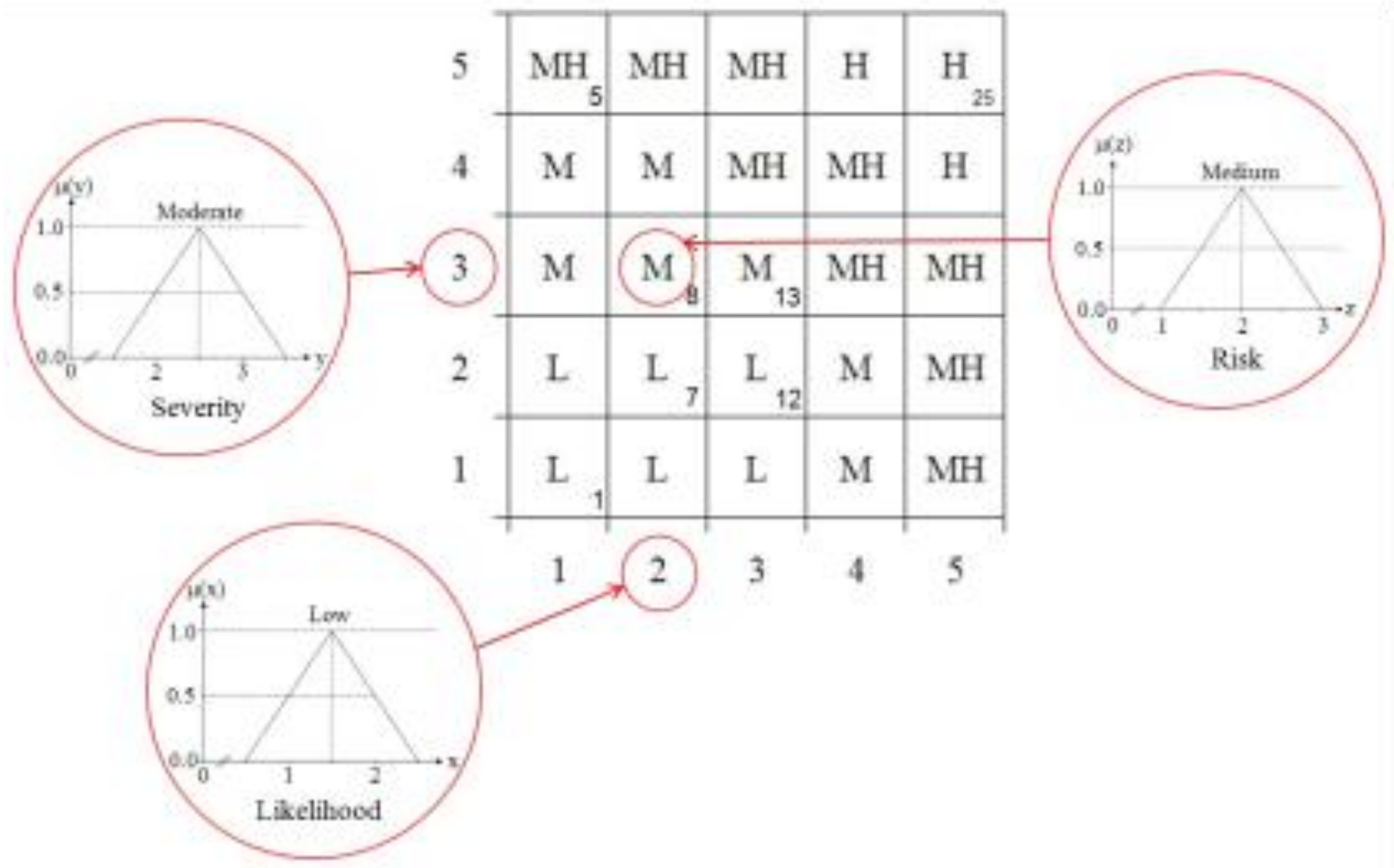
Severity categories:
1. negligible
2. low
3. moderate
4. high
5. catastrophic

Likelihood categories:
1. very low
2. low
3. moderate
4. high
5. very high

Rule 5: IF Likelihood is "Very Low" and Severity is "Catastrophic" THEN the risk is "medium high"

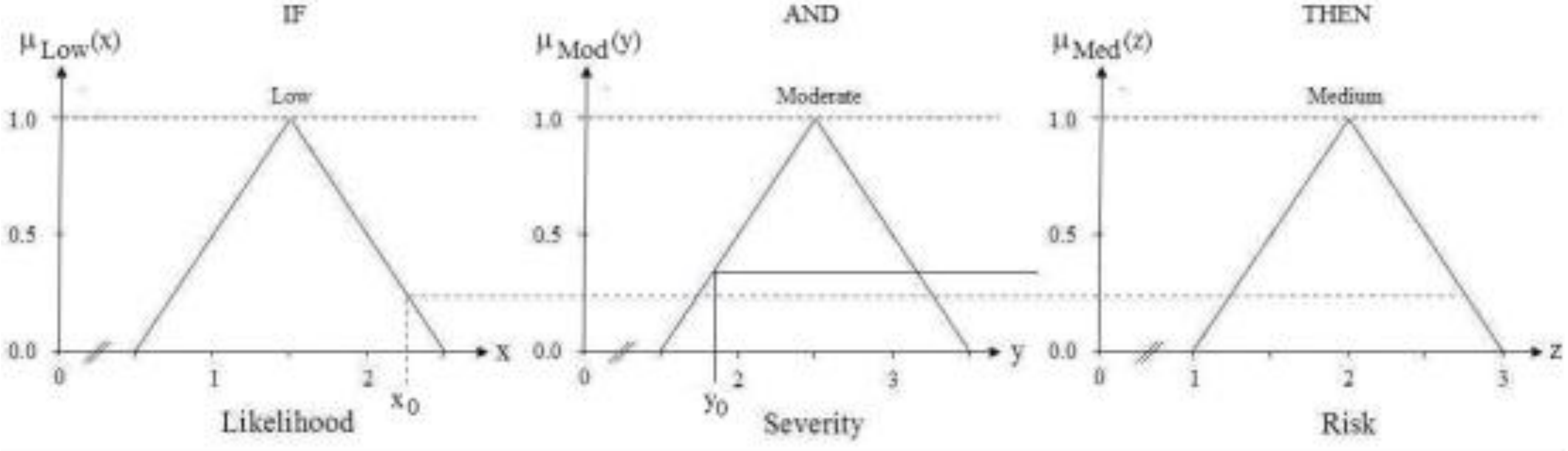
Rule 8: IF Likelihood is "Low" and Severity is "Moderate" THEN the risk is "medium"

Short Note on the (Fuzzy) Risk Matrix



Rule 8: IF Likelihood is "Low" and Severity is "Moderate" THEN the risk is "medium"

Short Note on the (Fuzzy) Risk Matrix



Rule 8: IF Likelihood is "Low" and Severity is "Moderate" THEN the risk is "medium"

$$m_{Med}(z) = \min\{m_{Low}(x_0), m_{Mod}(y_0)\}$$